Model for winning
Commonly winning probability for player a of game between players a and b is modeled as

$$P=\frac{1}{1+e^{k(S_a-S_b)}}$$

where S_a and S_b are players playing strengths, and k is multiplier that depends on those.

This equation is used in multiple ways

- When modeling (or predicting) game results and player strengths.
- When looking at actual games and using rankings as strengths.

GOR model sets $k=100/a$. In GOR system a has values from 70 to 200, corresponding to k values from 1.43 to 0.5.

One true k?
Now there is belief that single, correct k (or a in GOR system) would exists, and this k should be applied to all of these situations. This k could depend on players strengths (or ratings), but this dependency would be same in all systems. So k is assumed to be same for (unknown) correct strength and ratings that deviate significantly from correct strengths.

Problem with belief
In following it is shown that even in simplified system this assumption is not correct and k depends on accuracy of used rating even in this simplified system.

Simplifications
In order to make this analysis easy to follow for non-mathematical people, following assumptions are made:

- k does not depend on player strengths, but is constant in system.
- Player ratings are integers, so only integer differences in player ratings do exists.

For the analysis following terms are fixed:

- Strength S is true strength of player. (In reality this is not known, but only estimates of different accuracy.)
- Rating R is (inaccurate) rating assigned to player.

Now for simplicity we restrict to situation where

$$R_n=S_n+E_n,\quad E_n=\begin{cases}0, P=1/3\\-1, P=1/3\\+1, P=1/3\end{cases}$$

that is error E_n has equal probability to be 0, -1 or +1.

If original assumption about k being independent of system is true, this restricted system should also have k independent of system.

Analysis

In order to show that k depends on system, it is sufficient to show that different winning probabilities are seen for one unit strength difference and one unit rating difference.

As k was assumed not to depend on strength, analysis for one unit differences is valid for all strengths.

Strength difference
Expected winning probability for strength difference of one unit is

$$P_{S1}=\frac{1}{1+e^{k}}$$

Rating difference
For rating system difference of one rating gives
$R_a-R_b=S_a+E_a-S_b-E_b=(S_a-S_b)+(E_a-E_b)=1$

So strength difference can be calculated as function of error difference:
$(S_a-S_b)=1-(E_a-E_b)$
Tabulating all possible values for E_a-E_b we get

E_a-E_b	-2	-1	0	+1	+2
$P(E_a-E_b)$	1/9	2/9	3/9	2/9	1/9
S_a-S_b	+3	+2	+1	0	-1
Probability of win	$\frac{1}{1+e^{3k}}$	$\frac{1}{1+e^{2k}}$	$\frac{1}{1+e^{k}}$	0.5	$\frac{1}{1+e^{-k}}$

Winning probability is weighted sum of probabilities:
$$P_{R1}=\frac{1/9}{1+e^{3k}}+\frac{2/9}{1+e^{2k}}+\frac{3/9}{1+e^{1k}}+2/9\,0.5+\frac{1/9}{1+e^{-1k}}$$

Comparing strength and rating
Now, if winning probabilities would be same in both systems, following should be true:

$$\frac{1}{1+e^{k}}\stackrel{?}{=}\frac{1/9}{1+e^{3k}}+\frac{2/9}{1+e^{2k}}+\frac{3/9}{1+e^{1k}}+2/9\,0.5+\frac{1/9}{1+e^{-1k}}$$

Which clearly is not true, (except when k=0). Systems produce different winning probabilities.

By assigning example value of k=1, one gets winning probabilities 0.268 and 0.313.

Comments
When system's rating values get more accurate, higher k value explains game results better using system's ratings.

So value of *k* explaining best game results reflects accuracy of rating used. Given sufficient caution, this can be used as rough indication of system's accuracy. However indication is not accurate and can mislead seriously.

Best *k* value for some system is different from *k* that best explain games. Ultimately best k value may require experimentation with actual data to be determined.