Ko-fight as a mathematical problem: when to end a ko-fight.

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"When to end a ko-fight" is considered to be a difficult question. In the book "Attack and Defense" Ishida and Davies give the advice "end the fight when this enables you to win the game". Often, it is difficult to estimate exact numerical values for the threats, or for the target of the fight. Towards the endgame, however, these values tend to become well defined. Then, the question of ending the fight becomes easy, as this article tries to show to you.

Consider the example of Fig. 1. This is an immediate ko, where black or white needs to ignore one ko threat in order to capture the group of the opponent. The values can be easily counted: if white takes, he gets 7 points. If black takes, he gets 22 points. The value of the ko is the sum of these, 29 points.

The possible outcomes of the ko fight are marked in Fig. 2. The scale at left indicates the result as seen by black. If white or black had no ko threats at all, then the result would be gain 22 or loss 7. These possible results are marked by broken lines in the diagram.

Usually, both sides have some ko threats available. Let's assume that they have threats with the following values:

black: 33, 23, 21, 20, 18, 16, 12, 10, ...
white: 32, 24, 18, 12, 12, 12, 10, 5, ...

First, we assume that both sides are going to use their threats in the order of decreasing magnitude. The first threat will be made by black, and is valued 33 points.

If white would decide to ignore this threat (to "accept the offer at this price") then the outcome would be plus 26 points for black: "black loses the ko" is -7, but black would gain 33 points from the threat. - This outcome is illustrated by black 1 in the diagram. - It is clear that white shall not pay this price, even if he had no threats available.

So, white makes the next threat, of value 32. If black would accept this price (=ignore the threat), then the outcome would be minus 10 points for black: this is illustrated by the white 2 in the figure. It is clear that black will not pay this price.

The third threat is made by black. If white should accept this price, the result would be plus 16 for black, as indicated by the black 3 in the diagram. Now, it is not any more so evident if white should accept this or not: Is this the best he can hope to get, or will there be still better offers coming? White is hoping for a highest possible mark in the figure, so the question for him is: will there later on be black marks higher up in the figure? If yes, then he must not accept this one. - We assume that white answers the threat, so that the fight continues.

The next possible outcome is indicated by the white 4 in the figure. Should black accept this? He is hoping for the lowest possible mark in the figure, so the question for him is: are there yet lower marks coming?

The next possible outcomes are indicated in the diagram by alternating black and white marks. The white ones are coming down, so the result for black is improwing. He will not select any one of the white 2, 4, or 6. On the other hand, black should not hope to get white 14, because the earlier results black 11 and black 13 are better for white, who accordingly would select either one of these, in order to prevent black from getting at white 14.

By similar reasoning, white will not accept the results marked by black 3, 5, 7, or 9. On the other hand, he can not hope to get at black 11, because white 10 is better for black, who will select this result: black will ignore the white ko threat number 10, of value 12, and kill the white group. - This result is correct for both sides.

The rule for ending the ko fight.

We see that the correct result is found in the intersection of the sequences of black and white marks. To be more pre-

cise:

"the n:th ko threat should be ignored, if the sum of the n:th and (n-1):th threats is larger than, but the sum of the n:th and (n+1):th threats is less(or equal) than the value of the ko".

We apply this rule to the example: The sum of 9th and 10th threats is 30, which is more than 29. The sum of 10th and 11th is 28, less than 29. Thus the tenth threath is the critical one, which should be ignored.

Further remarks.

1. The rule is symmetric: it is the same for the player who is going to loose points in the ko fight, and for the player who is going to profit.
2. The values for thblack profit or white profit (22 and 7 in the example) are not needed in the rule which tells the correct moment for ending the ko fight. Only the sum (29) is needed. - I feel that this is somewhat surprising. - Of course, the outcome of the fight (who profits and how much) depends on the values of black and white profit. Ideally, one should estimate the outcome before starting the ko at all.
3. In real games, one often has several threats of the same numerical value (e.g. several moves in a life-or-death situation). In the example, the white threats numbered 8, 10, and 12 all are of value 12. We see that even if white did not possess his largest threat (value 32) at all, the outcome of the fight would nevertheless be the same: the white marks would be moved one step to the left in the diagram, but the intersection point would be in the same height as before. In such a situation, a player might well leave one of his large ko threats unplayed, either because he wants to spare it for a later possible ko fight, or because the move has undesirable after effects (aji).
4. For the approach-move ko fight, a more complex diagram might be constructed. This is, however, left to a possible later article.
5. In the example, it would also be correct for black to

ignore the white threat number 8. This would lead to the same end result as playing according to our rule.

6. Very well, if this article has made clear the mathematical reasoning in a ko fight. Let us not, however, imagine that we are now masters of ko fights. There are enough of difficulties left in the estimation of the values of the threats, in the estimation of bad after-effects, and in playing situations so that you get the largest possible number of ko threats, or limit the opponent to the least possible number of threats.

Fig. 1.

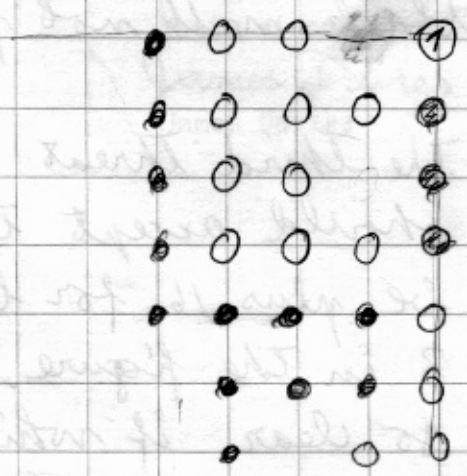
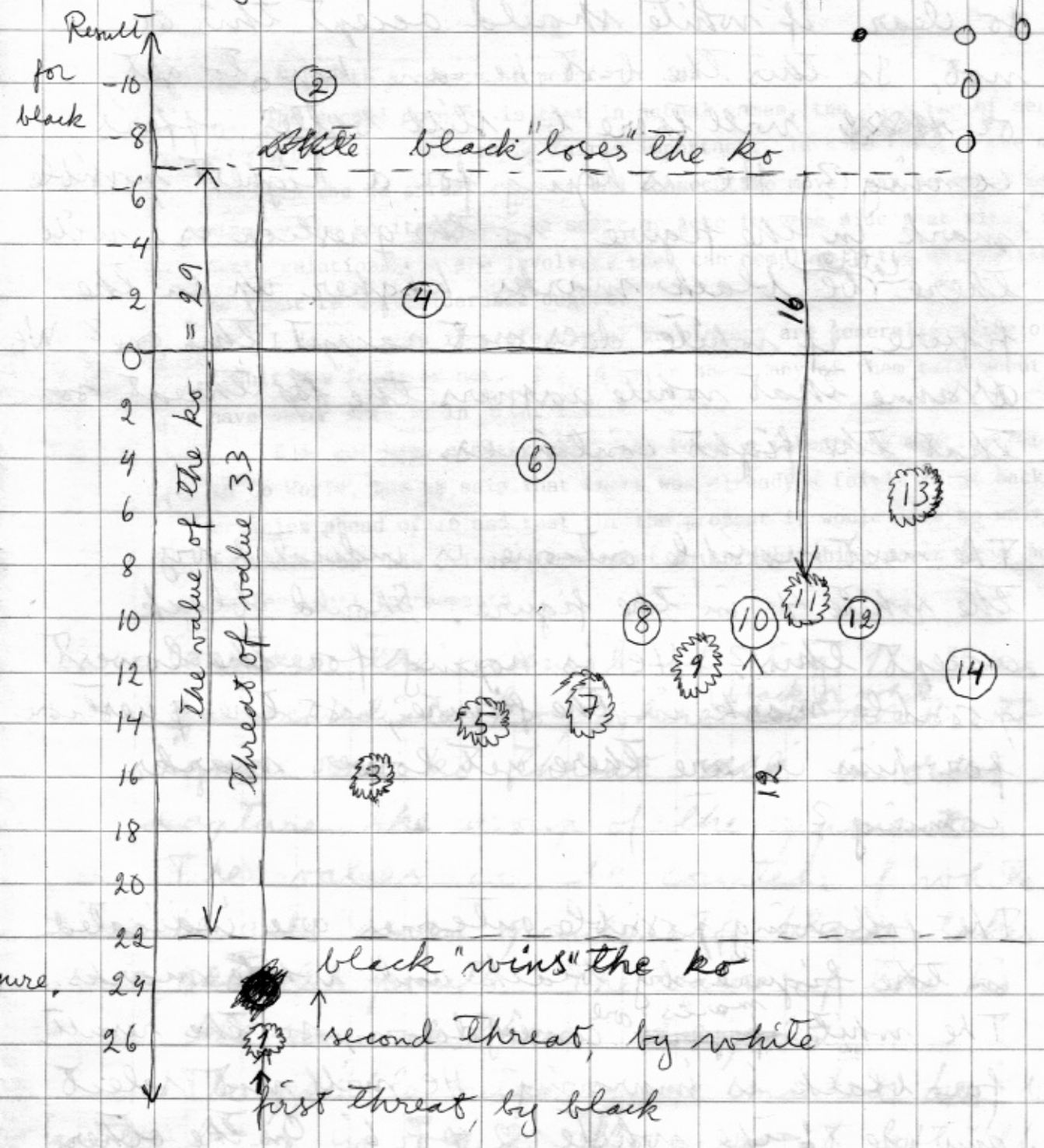


Fig. 2.


Result for black
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
20
22
24
26
~~White~~ black "loses" the ko
The value of the ko = 29
Threat of value 33
16
18
black "wins" the ko
second threat, by white
first threat, by black